Light Quenched Hadron Spectrum and Decay Constants on different Lattices

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We present a study of $\mathcal{O}(2000)$ (quenched) lattice configurations from the APE collaboration, for $6.0 \leq \beta \leq 6.4$ using both the Wilson and the SW-Clover fermion action. We determine the light hadronic spectrum and meson decay constants. We extract the inverse lattice spacing using data at the *simulated values* of the quark mass. We find an agreement with the experimental data of $\sim 5\%$ for mesonic masses and $\sim 10\% - 15\%$ for baryonic masses and pseudoscalar decay constants. A larger deviation is present for the vector decay constants.

1. INTRODUCTION

In the last four years the Ape group has been extensively studying lattice QCD in the quenched approximation. Several simulations have been done to study weak matrix elements such as f_D, f_B, B_K and to study semileptonic decays [1]-[5]. These simulations have allowed to study the dependence of the results on the spacing a and to investigate finite size effects. Here we present results for light meson masses and decay constants and baryon spectroscopy. The results have been obtained from eight sets of data at $\beta = 6.0$, 6.2 and 6.4 using either the Wilson action or the "improved" SW-Clover action, in order to reduce $O(a)$ effects. The parameters used in each simulation are listed in Table 2. The Ape group has performed extensive comparisons on data extracted from smeared and non-smeared propagators and found no real improvement for lattices with a time extent of 64 at $\beta = 6.0$ and $\beta = 6.2$ [3]. Note that the simulations have been performed at β values of 6.0 or larger. This is to negate the large systematic error present in lattice data for $\beta \lesssim 6.0$ due to lattice artifacts [6]. All the results we obtained will be described in greater detail in a forthcoming paper [7].

*presented by L. Giusti

Table 1
Predicted meson masses in GeV for all lattices.

	M_ρ	$M_{\eta'}$	M_ϕ
Exper.	0.770	0.686	1.019
Lat I	0.809(7)	0.6849(3)	0.977(7)
Lat II	0.808(3)	0.6849(1)	0.978(3)
Lat III	0.81(1)	0.6849(5)	0.98(1)
Lat IV	0.803(6)	0.6851(2)	0.984(6)
Lat V	0.79(1)	0.6856(5)	1.00(1)
Lat VI	0.797(7)	0.6853(3)	0.989(7)
Lat VII	0.796(4)	0.6853(2)	0.990(4)
Lat VIII	0.792(4)	0.6855(2)	0.994(4)

2. MESON MASSES AND DECAY CONSTANTS

Meson masses and decay constants have been extracted from two-point correlation functions of the following local operators

$$P_5(x) = i\bar{q}(x)\gamma_5 q(x), \quad V_k(x) = \bar{q}(x)\gamma_k q(x) \ ,$$

$$A_\mu(x) = \bar{q}(x)\gamma_\mu\gamma_5 q(x)$$

in the standard APE way [7]. We fit the correlation functions of these operators to a single par-

Table 2
Summary of the parameters of the runs analyzed in this work and time windows used in the fits.

	Lat I	Lat II	Lat III	Lat IV	Lat V	Lat VI	Lat VII	Lat VIII
Ref	[1]	[1,2]	[4]	[4]	[5]	[2,3]	[4]	[4]
β	6.0	6.0	6.2	6.2	6.2	6.2	6.4	6.4
Action	SW	Wil	SW	Wil	SW	Wil	Wil	SW
# Confs	200	320	250	250	200	110	400	400
Volume	$18^3 \times 64$	$18^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$18^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$	$24^3 \times 64$
K	-	-	0.14144	0.1510	-	-	0.1488	0.1400
	0.1425	0.1530	0.14184	0.1515	0.14144	0.1510	0.1492	0.1403
	0.1432	0.1540	0.14224	0.1520	0.14190	0.1520	0.1496	0.1406
	0.1440	0.1550	0.14264	0.1526	0.14244	0.1526	0.1500	0.1409
				Mesons with zero momentum				
$t_1 - t_2$	15-28	15-28	18-28	18-28	18-28	18-28	24-30	24-30
				Baryons with zero momentum				
$t_1 - t_2$	12-21	12-21	18-28	18-28	18-28	18-28	22-28	22-28

ticle propagator with a *sinh* in the case of axial-pseudoscalar function and with a *cosh* in other cases. The pseudoscalar decay constant f_{PS} has been extracted by combining the fit of $\langle A_0 P_5 \rangle$ with the ratio $\langle A_0 P_5 \rangle / \langle P_5 P_5 \rangle$. The errors have been estimated by a jacknife procedure, blocking the data in groups of 10 configurations and we have checked that there are no relevant changes in the error estimate by blocking groups of configurations of different size. We have fitted the correlation functions in time windows reported in Table 2. The time fit intervals are chosen with the following criteria: we fix the lower limit of the intervals as the one at which there is a stabilization of the effective mass and as the upper limit the furthest possible point before the error overwhelms the signal. We discard the possibility of fitting in a restricted region where a plateau is present, as the definition of such a region is highly questionable [8]. For lattices with highest number of configurations, i.e. LatII, LatVII and LatVIII, we confirm that higher statistics do not lead to a longer or better (relative to the statistical error) plateau [8]. Once the hadronic correlation functions have been fitted and the lattice masses and matrix elements extracted, we extract as much physics as possible from the "strange" region so that the chiral extrapolation will be needed only in few cases. The method we use is outlined below:

Table 3
Extrapolated/interpolated meson decay constants

	$\dfrac{f_\pi}{Z_A m_\rho}$	$\dfrac{1}{f_\rho Z_V}$	$\dfrac{f_K}{Z_A m_{K^*}}$	$\dfrac{1}{f_{K^*} Z_V}$
Lat I	0.17(1)	0.42(3)	0.172(9)	0.39(2)
Lat II	0.25(1)	0.51(2)	0.239(8)	0.48(2)
Lat III	0.16(1)	0.39(3)	0.164(9)	0.36(2)
Lat IV	0.21(1)	0.47(2)	0.214(8)	0.45(1)
Lat V	0.19(3)	0.30(4)	0.18(2)	0.30(3)
Lat VI	0.21(1)	0.49(3)	0.21(1)	0.46(2)
Lat VII	0.23(2)	0.39(2)	0.22(1)	0.38(1)
Lat VIII	0.19(1)	0.30(2)	0.18(1)	0.29(1)

- We define the lattice planes for meson masses and decay constants ($M_V a$, $(M_{PS} a)^2$), ($f_{PS} a / Z_A$, $(M_{PS} a)^2$) and ($1/(f_V Z_V)$, $(M_{PS} a)^2$) where the subscripts PS and V stand for pseudoscalar and vector meson. We plot the Monte Carlo data for each kappa used in the simulation on these planes;
- On the vector meson plane ($M_V a$, $(M_{PS} a)^2$) we impose the physical ratios M_{K^*}/M_K, M_ρ / M_π and find the values of $M_\pi a$, $M_\rho a$ (only one independent), $M_K a$, $M_{K^*} a$ (only one independent), $M_{\eta'} a$ and $M_\phi a$;
- We now use the value of meson masses determined above to read off the lattice meson decay constants, $(f_\pi a / Z_A)$, $(f_K a / Z_A)$, $(f_\rho Z_V)^{-1}$ and

$(f_{K^*}Z_V)^{-1}$ from the corresponding f_{PS} and f_V planes.

This procedure to extract physical quantities only requires meson masses and not unphysical quantities such as quark masses or k values. It allows us to study the *strange* physics and fix the lattice spacing directly in the region where data have been simulated without chiral extrapolation to zero quark mass. This approach therefore reduces the errors on physical quantities induced by the chiral extrapolation. Using the values of a^{-1} from M_{K^*} we have obtained the physical value in GeV of meson masses reported in Table 1. Comparing the vector meson mass (in lattice units) from lattices LatIII, LatV and [9] we infer that there is the possibility of some residual finite volume effects on the 18^3 lattice at $\beta = 6.2$ [7]. This problem may also be present in our $\beta = 6.4$ data for which the physical volume is the same as in Lat V. Further investigations at larger lattice sizes are necessary to make the situation clearer. Turning to the continuum limit, any dependence of the meson spectrum on a is small and difficult to interpret unambiguously at this stage. In Table 3 we report results for the meson decay constants without including the renormalization constants Z_V and Z_A. For both the pseudoscalar and vector decay constants we notice a difference between the Wilson and SW-Clover data. This is presumably due to the different renormalization constants and the smaller $O(a)$ effects in the latter case. There may also be a small residual finite lattice spacing effect in the vector decay constant in the SW-Clover data which needs further study. Overall our results agree with experimental data to $\sim 5\%$ for meson spectrum and to $\sim 10\% - 15\%$ for the pseudoscalar decay constants. In our opinion the vector decay constant deserves a much more careful study at larger volume and β.

3. BARYON MASSES

Baryon masses have been extracted from two-point correlation functions of the following local operators

$$
\begin{aligned}
N &= \epsilon_{abc}(u^a C\gamma_5 d^b)u^c \\
\Delta_\mu &= \epsilon_{abc}(u^a C\gamma_\mu u^b)u^c
\end{aligned}
$$

Table 4
Predicted baryon masses in GeV for all lattices.

	M_N	$M_{\Lambda\Sigma}$	M_Ξ	M_Δ	M_Ω
Exper.	0.9389	1.135	1.3181	1.232	1.6724
Lat I	1.09(5)	1.21(4)	1.32(4)	1.3(1)	1.60(9)
Lat II	1.19(5)	1.29(4)	1.40(4)	1.46(7)	1.71(4)
Lat III	1.1(1)	1.22(8)	1.34(7)	-	-
Lat IV	1.17(7)	1.28(6)	1.39(5)	-	-
Lat V	1.1(2)	1.2(2)	1.4(1)	1.6(3)	1.9(2)
Lat VI	1.2(1)	1.3(1)	1.40(9)	1.50(9)	1.72(5)
Lat VII	1.21(9)	1.32(8)	1.43(6)	1.4(2)	1.72(9)
Lat VIII	1.2(1)	1.29(8)	1.41(7)	1.3(2)	1.7(1)

in the standard way by fitting the two point correlation functions to a single particle propagator with an *exp* function. The errors have been estimated as in the meson case. We have used the value of meson masses to read off the lattice baryon masses from the planes $(M_N a, (M_{PS}a)^2)$ and $(M_\Delta a, (M_{PS}a)^2)$. Using the same values of a^{-1} used for meson masses we have obtained the results reported in Table 4. For baryons we find very good agreement with the old APE [10] data, while we find slightly larger values when comparing with JLQCD [8] and LANL [11]. Also for baryon masses we can conclude that we do not see a dependence on a and that we have an agreement with the experimental data of $\sim 10\% - 15\%$.

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